

**KIAS**  
**East Asia Symplectic Conference 2011**  
**Jun 21~25, 2011**

**Title and Abstract**

**Kwok Wai Chan (IPMU, Univ. of Tokyo)**

Title: Open Gromov–Witten invariants and mirror symmetry for toric manifolds

Abstract:

The mirror of a compact toric Kahler manifold is given by a Landau–Ginzburg model, i.e. a holomorphic function on a noncompact Kahler manifold. The function is usually called the superpotential and can be expressed in terms of open Gromov–Witten invariants defined by Fukaya–Oh–Ohta–Ono. In this talk, I will explain a formula which computes open Gromov–Witten invariants for certain classes of toric manifolds. As an application, we compute explicitly the mirror superpotentials for semi–Fano toric surfaces.

**Bohui Chen (Sichuan Univ.)**

Title: Extremal metrics on toric manifolds

Abstract:

Extremal metric on Kahler manifolds is a canonical metric proposed by E. Calabi. However, there has been few results on the existence. Donaldson initiated an important program to understand the extremal metrics on toric manifolds. In this talk, I will explain our works (joint with A. Li and L. Sheng) on this topic.

**Wan Keng Cheong (National Cheng Kung Univ.)**

Title: Quantum cohomology and the SYM–HILB correspondence

Abstract:

I will describe the SYM–HILB correspondence which relates the orbifold quantum cohomology of the symmetric product of a smooth toric surface  $S$  to the quantum cohomology of the Hilbert scheme of points in  $S$ . I will also strengthen the correspondence for the case where  $S$  is the minimal resolution of the cyclic quotient singularity  $C^2/Z_{r+1}$ . These results will give an affirmative answer to the crepant resolution conjecture.

**Yunhyung Cho (KAIST)**

Title: The Log-concavity of the Duistermaat-Heckman measure on hamiltonian  $S^1$ -manifolds.

ABSTRACT. Consider a closed Hamiltonian  $S^1$ -manifold and let  $\mu : M \rightarrow \mathbb{R}$  be a corresponding moment map. The Duistermaat-Heckman function  $DH : \mathbb{R} \rightarrow \mathbb{R}$  is defined as the symplectic volume of the reduced space, i.e.  $DH(t) := \int_{M_t} \omega_t^{n-1}$  where  $\omega_t$  is the reduced symplectic form on  $M_t = \mu^{-1}(t)/S^1$ . It is well-known that  $DH$  is log-concave when  $(M, \omega)$  is Kähler, or  $M$  is a co-adjoint orbit of a compact Lie group. In this talk, we prove that if the action is semifree and the fixed point set consists of isolated fixed points, then the Duistermaat-Heckman function is log-concave. With the same assumption, we also prove that any reduced symplectic form satisfies the hard Lefschetz property.

**Wu-Yen Chuang (National Taiwan Univ.)**

Title: Higher Rank Donaldson-Thomas Invariants and Wallcrossing

Abstract:

I will introduce ADHM sheaf theory, which is an alternative construction of the Donaldson-Thomas theory on local curve geometries. Applying wallcrossing formula we obtained the higher rank invariants with nontrivial curve classes. Other applications of the quiver sheaf theory, such as the computation of the Poincare polynomials of the Hitchin moduli, will also be discussed if time permits.

**Huijun Fan (Peking Univ.)**

Title: Schroedinger equation, deformation theory and LG B model

Abstract:

Let  $(M, g)$  be a complete noncompact Kaehler manifold with bounded geometry and  $f$  be a strongly tame holomorphic function defined on  $M$ . We can associate a twisted Laplacian operator to  $(M, g, f)$ . This is a Schroedinger type operator acting on forms. We can prove it has discrete spectrum in  $L^2$  space. As a consequence, we can construct the Hodge theory to  $(M, g, f)$ . If  $f_t$  is a strong deformation of  $f$ , then we can get the Hodge bundle over the parameter space and prove that it has  $\text{Stt}^*$  geometric structure. This gives a rigorous mathematical proof to Cecotti-Vafa's  $\text{Stt}^*$  geometry structure and furthermore, more interesting structures were found by us. This work can be viewed as the construction of Landau-Ginzburg B model.

**Nan-Kuo Ho (National Tsing Hua Univ.)**

Title: Morse theory and the moduli space of flat connections over a nonorientable surface.

Abstract:

We studied the moduli space of flat connections over a nonorientable surface via a Morse theory approach adapted from Atiyah and Bott's work. We defined a Yang-Mills functional on the space of all connections over a nonorientable surface and obtained a Morse stratification of the space. We defined super central extension of the fundamental group of the surface. The representation varieties defined using super central extension correspond to gauge equivariant Yang-Mills connections and enable us to obtain reduction formulas for equivariant strata. To describe the phenomenon of the stratification here, we defined anti-perfect Morse stratification, which is the case when the discrepancy (i.e. the difference between Morse series and Poincare series) reaches its maximal possible value while the perfect Morse stratification is when the discrepancy reaches its minimal possible value zero. This is a joint work with Chiu-Chu Melissa Liu.

**Hansol Hong (Seoul National Univ.)**

Title: ORBIFOLD MORSE-SMALE-WITTEN COMPLEX

Abstract:

We construct Morse-Smale-Witten complex for an effective orientable orbifold. We show that certain type of critical points of a Morse function has to be discarded to construct such a complex, and gradient flows should be counted with suitable weights. The homology of these complexes are shown to be isomorphic to the singular homology of the quotient spaces under the self-indexing assumptions.

**Kei Irie (Kyoto Univ.)**

Title: Displacement energy of unit cotangent bundles

Abstract:

We study the displacement energy of the unit cotangent bundle of a Riemannian manifold, when the manifold has non-empty boundary.

Our main result gives a nice upper bound of the displacement energy. As an application, we prove the existence of a short periodic billiard trajectory on the manifold.

**Hiroshi Iriyeh (Tokyo Denki Univ.)**

Title: Lagrangian Floer homology of a pair of real forms in Hermitian symmetric spaces of compact type

Abstract:

In this talk we calculate the Lagrangian Floer homology  $\mathcal{HF}(L_0, L_1 : \{\mathbb{W}\mathbb{m}\mathbb{a}\mathbb{t}\mathbb{h}\mathbb{b}\mathbb{b}\ Z\}_2)$  of a pair of real forms  $(L_0, L_1)$  in a monotone Hermitian symmetric space  $M$  of compact type in the case where  $L_0$  is not necessarily congruent to  $L_1$ . In particular, we have a generalization of the Arnold–Givental inequality in the case where  $M$  is irreducible. The notion of antipodal sets in symmetric space theory plays an important role in the calculation.

**Jian Zhou (Tsinghua Univ.)**

TBA

**Jinhong Kim (KAIST)**

Title: Rigidity of Hamiltonian circle actions with the minimality conditions

Abstract:

Let  $M$  be a compact symplectic manifold with a Hamiltonian circle action and the moment map  $f$ . Then we can consider the minimality conditions about the sum of the dimensions of the fixed point sets and the even Betti numbers of  $M$  which are called the minimal dimension condition and the minimal even Betti number condition. Since the moment map  $f$  is a perfect Morse–Bott function, it can be shown that if  $M$  satisfies the minimal even Betti number condition, then the Hamiltonian circle action satisfies the minimal dimension condition. However, the converse is not true, in general. In this talk, I plan to discuss some rigidity theorems of Hamiltonian circle actions under such minimality conditions as well as other related questions.

**Siu Cheong Lau (The Chinese Univ. of Hong Kong)**

Title: Open GW invariants and mirror maps for toric CY manifolds

Abstract:

Mirror symmetry exchanges symplectic geometries of a Kaehler manifold with complex geometries of its mirror. In particular, the (complexified) Kaehler moduli is locally identified with the complex moduli of the mirror (around the large complex structure limit) via the so-called mirror map, whose geometric meaning was unclear.

In this talk, I will consider the mirror maps for toric CY manifolds and show that they contain symplectic enumerative information, which are the one-pointed open Gromov-Witten invariants of the toric fibers.

**JaeHyoun Lee (Ewha Womans Univ.)**

Title: Geometry of Symplectic Subspaces

Abstract:

We consider symplectic subspaces in the context of linear and nonlinear setup which are symplectic Grassmanians and symplectic knot spaces respectively. In both of the cases, we introduce symplectic structures and show that the structures can be also obtained by the symplectic quotient method. Furthermore, we study the correspondence between the coisotropic subspaces to Lagrangian geometry in both cases, and also relate almost complex subspaces to holomorphic curves after introducing complex structures for both cases.

**Takeo Nishinou (Tohoku Univ.)**

Title: Counting curves via degeneration.

Abstract:

A suitable polyhedral decomposition of  $\mathbb{R}^n$  gives a toric degeneration of a toric variety. A tropical curve is related to a toric degeneration of a toric variety, by regarding it as a subset of the 1-skeleton of the polyhedral decomposition.

Using this correspondence, it is possible to count holomorphic curves in a toric variety by counting tropical curves. We generalize this to not necessarily toric cases, and also mention some applications.

**Siye Wu (The Univ. of Hong Kong)**

Title: Quantisation of the cotangent bundle of Lie groups

Abstract:

We consider a family of adapted complex structures on the cotangent bundle of a Lie group and find the BKS pairing relating the corresponding half-form quantisation. We show that the resulting bundle of quantum Hilbert spaces over the space of polarisations is flat. The vertical polarisation as a limit of complex polarisations yields the coherent state transform (or the Segal–Bargmann–Hall transform). We show that there is another limit of the complex polarisations that corresponds to the Peter–Weyl theorem. This is a joint work with W. Kirwin.

**Fabian Ziltener (KIAS)**

Title: A Quantum Kirwan Map and Symplectic Vortices

Abstract:

Given a Hamiltonian Lie group action on a symplectic manifold, I will explain how to construct a quantum deformation of the Kirwan map. It is defined by counting solutions of the symplectic vortex equations over the real plane. These equations are a gauge theoretical deformation of the Cauchy–Riemann equations. Based on this map, I will formulate a quantum abelianization conjecture (joint work with Chris Woodward).